Derivování bez úpravy

1. $y = \sqrt[3]{\ln(3x-x^2)} \;\; -> \;\; y = \left(\ln(3x-x^2)\right)^{\frac{1}{3}}$

$$y' = \frac{1}{3}\left(\ln(3x-x^2)\right)^{-\frac{2}{3}} \cdot \left(\ln(3x-x^2)\right)' =$$

$$= \frac{1}{3}\left(\ln(3x-x^2)\right)^{-\frac{2}{3}} \cdot \frac{1}{3x-x^2} \cdot (3x-x^2)' =$$

$$= \underline{\underline{\frac{1}{3}\left(\ln(3x-x^2)\right)^{-\frac{2}{3}} \cdot \frac{1}{3x-x^2} \cdot (3-2x)}}$$

2. $y = \dfrac{1}{\ln\left(\frac{x}{x^2-5}\right)} \longrightarrow y = \left(\ln\left(\frac{x}{x^2-5}\right)\right)^{-1}$

$$y' = -1 \cdot \left(\ln\left(\frac{x}{x^2-5}\right)\right)^{-2} \cdot \left(\ln\left(\frac{x}{x^2-5}\right)\right)' =$$

$$= -\left(\ln\left(\frac{x}{x^2-5}\right)\right)^{-2} \cdot \frac{1}{\frac{x}{x^2-5}} \cdot \left(\frac{x}{x^2-5}\right)' =$$

$$= \underline{\underline{-\left(\ln\left(\frac{x}{x^2-5}\right)\right)^{-2} \cdot \frac{x^2-5}{x} \cdot \frac{1\cdot(x^2-5) - x\cdot 2x}{(x^2-5)^2}}}$$

3. $y = \sin\left(\frac{2x}{3}+\pi\right) \cdot \sqrt{\cos\left(\frac{2}{3x}\right)}$

$$\left(\sin\left(\frac{2x}{3}+\pi\right)\right)' = \cos\left(\frac{2x}{3}+\pi\right) \cdot \left(\frac{2x}{3}+\pi\right)' = \cos\left(\frac{2x}{3}+\pi\right) \cdot \frac{2}{3}$$

$$\left(\sqrt{\cos\left(\frac{2}{3x}\right)}\right)' = \left(\left(\cos\left(\frac{2}{3x}\right)\right)^{\frac{1}{2}}\right)' =$$

$$= \frac{1}{2}\left(\cos\left(\frac{2}{3x}\right)\right)^{-\frac{1}{2}} \cdot \left(\cos\left(\frac{2}{3x}\right)\right)' = \frac{1}{2}\left(\cos\left(\frac{2}{3x}\right)\right)^{-\frac{1}{2}} \cdot \left(-\sin\left(\frac{2}{3x}\right)\right) \cdot \left(\frac{2}{3x}\right)' =$$

$$= \frac{1}{2}\left(\cos\left(\frac{2}{3x}\right)\right)^{-\frac{1}{2}} \cdot \left(-\sin\left(\frac{2}{3x}\right)\right) \cdot \left(-\frac{2}{3}x^{-2}\right)$$

$$\left[\frac{2}{3x} = \frac{2}{3}\cdot x^{-1}\right]$$

$$y' = \cos\left(\frac{2x}{3}+\pi\right) \cdot \frac{2}{3} \cdot \sqrt{\cos\left(\frac{2}{3x}\right)} +$$

$$\underline{\underline{+ \sin\left(\frac{2x}{3}+\pi\right) \cdot \frac{1}{2}\left(\cos\left(\frac{2}{3x}\right)\right)^{-\frac{1}{2}} \cdot \left(-\sin\left(\frac{2}{3x}\right)\right) \cdot \left(-\frac{2}{3}x^{-2}\right)}}$$

4. $y = x e^{1-x^2} \cos\frac{x}{5}$

$$y' = \left(x e^{1-x^2}\right)' \cdot \cos\frac{x}{5} + x e^{1-x^2} \cdot \left(\cos\frac{x}{5}\right)'$$

$$\left(x \cdot e^{1-x^2}\right)' = x' \cdot e^{1-x^2} + x \cdot \left(e^{1-x^2}\right)' =$$
$$= 1 \cdot e^{1-x^2} + x e^{1-x^2} \cdot (1-x^2)' =$$
$$= e^{1-x^2} + x e^{1-x^2} \cdot (-2x)$$

$$\left(\cos\frac{x}{5}\right)' = \left(-\sin\frac{x}{5}\right) \cdot \left(\frac{x}{5}\right)' = \left(-\sin\frac{x}{5}\right) \cdot \frac{1}{5}$$

$$\underline{\underline{y' = \left(e^{1-x^2} + x e^{1-x^2}(-2x)\right) \cdot \cos\frac{x}{5} + x e^{1-x^2}\left(-\sin\frac{x}{5}\right) \cdot \frac{1}{5}}}$$

5. $y = \dfrac{e^{-\frac{1}{x^3}}}{(\sin x)^2}$

$$y' = \frac{\left(e^{-\frac{1}{x^3}}\right)' \cdot (\sin x)^2 - e^{-\frac{1}{x^3}} \cdot \left((\sin x)^2\right)'}{(\sin x)^4}$$

$$\left(e^{-\frac{1}{x^3}}\right)' = e^{-\frac{1}{x^3}} \cdot \left(-\frac{1}{x^3}\right)' = e^{-\frac{1}{x^3}} \cdot \left(-x^{-3}\right)' =$$
$$= e^{-\frac{1}{x^3}} \cdot (-1) \cdot (-3) \cdot x^{-4} = 3 e^{-\frac{1}{x^3}} \cdot x^{-4}$$

$$\left((\sin x)^2\right)' = 2 \cdot \sin x \cdot (\sin x)' = 2 \sin x \cos x$$

$$\underline{\underline{y' = \frac{3 e^{-\frac{1}{x^3}} \cdot x^{-4} \cdot (\sin x)^2 - e^{-\frac{1}{x^3}} \cdot 2 \sin x \cos x}{(\sin x)^4}}}$$

6. $y = \operatorname{arctg}\left(\sqrt{1+x^2}\right)$

$$y' = \frac{1}{1 + \left(\sqrt{1+x^2}\right)^2} \cdot \left(\left(1+x^2\right)^{\frac{1}{2}}\right)' = \frac{1}{2+x^2} \cdot \frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}} \cdot 2x$$