1. Napište **Taylorův rozvoj 4. stupně** funkce $y = \dfrac{1}{\sqrt{x}}$ v bodě $x_0 = 1$.

Řešení: $T(x) : y = y(x_0) + \frac{1}{1!} \cdot y'(x_0) \cdot (x-1) + \frac{1}{2!} \cdot y''(x_0) \cdot (x-1)^2 +$
$+ \frac{1}{3!} \cdot y'''(x_0) \cdot (x-1)^3 + \frac{1}{4!} \cdot y^{(4)}(x_0) \cdot (x-1)^4 + \frac{1}{5!} \cdot y^{(5)}(x_0) \cdot (x-1)^5 + \dots$

$y(1) = x^{-\frac{1}{2}} = 1$
$y'(1) = -\frac{1}{2} \cdot x^{-\frac{3}{2}} = -\frac{1}{2}$
$y''(1) = \frac{3}{2 \cdot 2} \cdot x^{-\frac{5}{2}} = \frac{3}{4}$
$y'''(1) = -\frac{3 \cdot 5}{4 \cdot 2} \cdot x^{-\frac{7}{2}} = -\frac{3 \cdot 5}{8}$
$y^{(4)}(1) = \frac{3 \cdot 5 \cdot 7}{8 \cdot 2} \cdot x^{-\frac{9}{2}} = \frac{3 \cdot 5 \cdot 7}{16}$

$$T(x) : y = 1 - \frac{1}{2} \cdot (x-1) + \frac{3}{8} \cdot (x-1)^2 - \frac{5}{16} \cdot (x-1)^3 + \frac{35}{128} \cdot (x-1)^4$$

2. Napište **Taylorův rozvoj 3. stupně** funkce $y = x \cdot \sin x$ v bodě $x_0 = \dfrac{\pi}{2}$.

Řešení: $T(x) : y = y(x_0) + \frac{1}{1!} \cdot y'(x_0) \cdot (x-1) + \frac{1}{2!} \cdot y''(x_0) \cdot (x-1)^2 +$
$+ \frac{1}{3!} \cdot y'''(x_0) \cdot (x-1)^3 + \frac{1}{4!} \cdot y^{(4)}(x_0) \cdot (x-1)^4 + \frac{1}{5!} \cdot y^{(5)}(x_0) \cdot (x-1)^5 + \dots$

$y(\frac{\pi}{2}) = x \cdot \sin x = \frac{\pi}{2} \cdot \underbrace{\sin \frac{\pi}{2}}_{1} = \frac{\pi}{2}$

$y'(\frac{\pi}{2}) = \sin x + x \cdot \cos x = \underbrace{\sin \frac{\pi}{2}}_{1} + \frac{\pi}{2} \cdot \underbrace{\cos \frac{\pi}{2}}_{0} = 1$

$y''(\frac{\pi}{2}) = \cos x + \cos x + x \cdot (-\sin x) = 2\cos x - x \cdot \sin x = 2 \underbrace{\cos \frac{\pi}{2}}_{0} - \frac{\pi}{2} \cdot \underbrace{\sin \frac{\pi}{2}}_{1} = -\frac{\pi}{2}$

$y'''(\frac{\pi}{2}) = -2\sin x - \sin x - x \cdot \cos x = -3\sin x - x \cdot \cos x = -3 \underbrace{\sin \frac{\pi}{2}}_{1} - \frac{\pi}{2} \cdot \underbrace{\cos \frac{\pi}{2}}_{0} = -3$

$$T(x) : y = \frac{\pi}{2} + \left(x - \frac{\pi}{2}\right) - \frac{\pi}{4} \cdot \left(x - \frac{\pi}{2}\right)^2 - \frac{1}{2} \cdot \left(x - \frac{\pi}{2}\right)^3$$

3. NapištE **Taylorův rozvoj 6. stupně** funkce $y = \sin 2x$ v bodě $x_0 = \dfrac{\pi}{4}$.

Řešení: $T(x) : y = y(x_0) + \frac{1}{1!} \cdot y'(x_0) \cdot (x-1) + \frac{1}{2!} \cdot y''(x_0) \cdot (x-1)^2 +$

$+ \frac{1}{3!} \cdot y'''(x_0) \cdot (x-1)^3 + \frac{1}{4!} \cdot y^{(4)}(x_0) \cdot (x-1)^4 + \frac{1}{5!} \cdot y^{(5)}(x_0) \cdot (x-1)^5 + \ldots$

$y(\frac{\pi}{4}) = \sin 2x = \underbrace{\sin \frac{\pi}{2}}_{1} = 1$ $\qquad y'(\frac{\pi}{4}) = 2\cos 2x = 2 \underbrace{\cos \frac{\pi}{2}}_{0} = 0$

$y''(\frac{\pi}{4}) = 2 \cdot 2 \cdot (-\sin 2x) = -4 \underbrace{\sin \frac{\pi}{2}}_{1} = -4$ $\qquad y'''(\frac{\pi}{4}) = -4 \cdot 2 \cdot \cos 2x = -8 \underbrace{\cos \frac{\pi}{2}}_{0} = 0$

$y^{(4)}(\frac{\pi}{4}) = -8 \cdot 2 \cdot (-\sin 2x) = 16 \underbrace{\sin \frac{\pi}{2}}_{1} = 16$ $\qquad y^{(5)}(\frac{\pi}{4}) = 16 \cdot 2 \cdot \cos 2x = 32 \underbrace{\cos \frac{\pi}{2}}_{0} = 0$

$y^{(6)}(\frac{\pi}{4}) = 32 \cdot 2 \cdot (-\sin 2x) = -64 \underbrace{\sin \frac{\pi}{2}}_{1} = -64$

$$T(x) : y = 1 - 2 \cdot \left(x - \frac{\pi}{4}\right)^2 + \frac{2}{3} \cdot \left(x - \frac{\pi}{4}\right)^4 - \frac{4}{45} \cdot \left(x - \frac{\pi}{4}\right)^6$$