

1. Napište **Taylorův rozvoj 4. stupně** funkce  $y = \frac{1}{\sqrt{x}}$  v bodě  $x_0 = 1$ .

**Řešení:**  $T(x) : y = y(x_0) + \frac{1}{1!} \cdot y'(x_0) \cdot (x - 1) + \frac{1}{2!} \cdot y''(x_0) \cdot (x - 1)^2 +$   
 $+ \frac{1}{3!} \cdot y'''(x_0) \cdot (x - 1)^3 + \frac{1}{4!} \cdot y^{(4)}(x_0) \cdot (x - 1)^4$

$$y(1) = x^{-\frac{1}{2}} = 1$$

$$y'(1) = -\frac{1}{2} \cdot x^{-\frac{3}{2}} = -\frac{1}{2}$$

$$y''(1) = \frac{3}{2 \cdot 2} \cdot x^{-\frac{5}{2}} = \frac{3}{4}$$

$$y'''(1) = -\frac{3 \cdot 5}{4 \cdot 2} \cdot x^{-\frac{7}{2}} = -\frac{3 \cdot 5}{8}$$

$$y^{(4)}(1) = \frac{3 \cdot 5 \cdot 7}{8 \cdot 2} \cdot x^{-\frac{9}{2}} = \frac{3 \cdot 5 \cdot 7}{16}$$

$$T(x) : y = 1 - \frac{1}{2} \cdot (x - 1) + \frac{3}{8} \cdot (x - 1)^2 - \frac{5}{16} \cdot (x - 1)^3 + \frac{35}{128} \cdot (x - 1)^4$$

2. Napište **Taylorův rozvoj 3. stupně** funkce  $y = x \cdot \sin x$  v bodě  $x_0 = \frac{\pi}{2}$ .

**Řešení:**  $T(x) : y = y(x_0) + \frac{1}{1!} \cdot y'(x_0) \cdot (x - x_0) + \frac{1}{2!} \cdot y''(x_0) \cdot (x - x_0)^2 +$   
 $+ \frac{1}{3!} \cdot y'''(x_0) \cdot (x - x_0)^3 + \frac{1}{4!} \cdot y^{(4)}(x_0) \cdot (x - x_0)^4 + \frac{1}{5!} \cdot y^{(5)}(x_0) \cdot (x - x_0)^5 + \dots$

$$y\left(\frac{\pi}{2}\right) = x \cdot \sin x = \frac{\pi}{2} \cdot \underbrace{\sin \frac{\pi}{2}}_1 = \frac{\pi}{2}$$

$$y'\left(\frac{\pi}{2}\right) = \sin x + x \cdot \cos x = \underbrace{\sin \frac{\pi}{2}}_1 + \frac{\pi}{2} \cdot \underbrace{\cos \frac{\pi}{2}}_0 = 1$$

$$y''\left(\frac{\pi}{2}\right) = \cos x + \cos x + x \cdot (-\sin x) = 2 \cos x - x \cdot \sin x = 2 \underbrace{\cos \frac{\pi}{2}}_0 - \frac{\pi}{2} \cdot \underbrace{\sin \frac{\pi}{2}}_1 = -\frac{\pi}{2}$$

$$y'''\left(\frac{\pi}{2}\right) = -2 \sin x - \sin x - x \cdot \cos x = -3 \sin x - x \cdot \cos x =$$

$$= -3 \underbrace{\sin \frac{\pi}{2}}_1 - \frac{\pi}{2} \cdot \underbrace{\cos \frac{\pi}{2}}_0 = -3$$

$$T(x) : y = \frac{\pi}{2} + \left(x - \frac{\pi}{2}\right) - \frac{\pi}{4} \cdot \left(x - \frac{\pi}{2}\right)^2 - \frac{1}{2} \cdot \left(x - \frac{\pi}{2}\right)^3$$

3. Napište **Taylorův rozvoj 6. stupně** funkce  $y = \sin 2x$  v bodě  $x_0 = \frac{\pi}{4}$

$$\text{\textbf{Řešení:}} \quad T(x) : y = y(x_0) + \frac{1}{1!} \cdot y'(x_0) \cdot (x - x_0) + \frac{1}{2!} \cdot y''(x_0) \cdot (x - x_0)^2 + \\ + \frac{1}{3!} \cdot y'''(x_0) \cdot (x - x_0)^3 + \frac{1}{4!} \cdot y^{(4)}(x_0) \cdot (x - x_0)^4 + \frac{1}{5!} \cdot y^{(5)}(x_0) \cdot (x - x_0)^5 + \dots$$

$$y\left(\frac{\pi}{4}\right) = \sin 2x = \underbrace{\sin \frac{\pi}{2}}_1 = 1$$

$$y'\left(\frac{\pi}{4}\right) = 2 \cos 2x = 2 \underbrace{\cos \frac{\pi}{2}}_0 = 0$$

$$y''\left(\frac{\pi}{4}\right) = 2 \cdot 2 \cdot (-\sin 2x) = -4 \underbrace{\sin \frac{\pi}{2}}_1 = -4$$

$$y'''\left(\frac{\pi}{4}\right) = -4 \cdot 2 \cdot \cos 2x = -8 \underbrace{\cos \frac{\pi}{2}}_0 = 0$$

$$y^{(4)}\left(\frac{\pi}{4}\right) = -8 \cdot 2 \cdot (-\sin 2x) = 16 \underbrace{\sin \frac{\pi}{2}}_1 = 16$$

$$y^{(5)}\left(\frac{\pi}{4}\right) = 16 \cdot 2 \cdot \cos 2x = 32 \underbrace{\cos \frac{\pi}{2}}_0 = 0$$

$$y^{(6)}\left(\frac{\pi}{4}\right) = 32 \cdot 2 \cdot (-\sin 2x) = -64 \underbrace{\sin \frac{\pi}{2}}_1 = -64$$

$$T(x) : y = 1 - 2 \cdot \left(x - \frac{\pi}{4}\right)^2 + \frac{2}{3} \cdot \left(x - \frac{\pi}{4}\right)^4 - \frac{4}{45} \cdot \left(x - \frac{\pi}{4}\right)^6$$