Parciální derivace funkce o více proměnných je její derivace vzhledem k jedné z těchto proměnných (derivujeme tuto funkci jako funkci jen ***jedné*** proměnné), kdy všechny ostatní proměnné pokládáme za konstanty.

Výsledkem parciální derivace je funkce. ***Tuto můžeme opět*** parciálně derivovat a dostáváme *parciální derivaci parciální derivace*, nebo-li druhou parciální derivaci (což je opět funkce). ***Tuto můžeme opět*** ... třetí parciální derivaci ...

1. $[x \in \mathbb{R};\ y \in \mathbb{R}]$ Napište všechny třetí parciální derivace funkce

$$f(x,y) = x^3 + y^2 - 6xy - 39x + 18y + 4$$

$f_x = 3x^2 - 6y - 39$	$f_{xx} = 6x$	$f_{xxx} = 6$, $f_{xxy} = 0$	Jsou-li ***smíšené derivace*** f_{xy} a f_{yx} v okolí nějakého bodu $\mathcal{A}$ ***spojité***, pak jsou si v tomto bodě $\mathcal{A}$ rovny. Nebo-li: $f_{xy}(\mathcal{A}) = f_{yx}(\mathcal{A})$.
	$f_{xy} = -6$	$f_{xyx} = 0$, $f_{xyy} = 0$	
$f_y = 2y - 6x + 18$	$f_{yx} = -6$	$f_{yxx} = 0$, $f_{yxy} = 0$	Jsou-li ***smíšené derivace*** f_{xxy}, f_{xyx} a f_{yxx} v okolí bodu $\mathcal{A}$ ***spojité***, pak jsou si v tomto bodě $\mathcal{A}$ rovny. $f_{xxy}(\mathcal{A}) = f_{xyx}(\mathcal{A}) = f_{yxx}(\mathcal{A})$ Podobně $f_{xyy}(\mathcal{A}), f_{yxy}(\mathcal{A}), f_{yyx}(\mathcal{A})$
	$f_{yy} = 2$	$f_{yyx} = 0$, $f_{yyy} = 0$	

2. $[x \in \mathbb{R};\ y \in \mathbb{R}]$ Napište všechny třetí parciální derivace funkce

$$f(x,y) = 6xy^2 - 2x^3 - 3y^3$$

$f_x = 6y^2 - 6x^2$
$f_y = 12xy - 9y^2$

$f_{xx} = -12x$
$f_{xy} = 12y$
$f_{yx} = 12y$
$f_{yy} = 12x - 18y$

$f_{xxx} = -12$
$f_{xxy} = 0$
$f_{xyx} = 0$
$f_{xyy} = 12$
$f_{yxx} = 0$
$f_{yxy} = 12$
$f_{yyx} = 12$
$f_{yyy} = -18$

3. [$x \in \mathbb{R}$; $y \in \mathbb{R}$] Napište všechny čtvrté parciální derivace funkce

$$f(x,y) = \mathrm{e}^{xy}$$

$$\begin{aligned}
f_x &= \mathrm{e}^{xy} \cdot y = y \cdot \mathrm{e}^{xy} \\
f_y &= \mathrm{e}^{xy} \cdot x = x \cdot \mathrm{e}^{xy}
\end{aligned}$$

$$\begin{aligned}
f_{xx} &= y \cdot \mathrm{e}^{xy} \cdot y = y^2 \cdot \mathrm{e}^{xy} \\
f_{xy} &= 1 \cdot \mathrm{e}^{xy} + y \cdot \mathrm{e}^{xy} \cdot x = (1 + xy) \cdot \mathrm{e}^{xy} \\
f_{yx} &= 1 \cdot \mathrm{e}^{xy} + x \cdot \mathrm{e}^{xy} \cdot y = (1 + xy) \cdot \mathrm{e}^{xy} \\
f_{yy} &= x \cdot \mathrm{e}^{xy} \cdot x = x^2 \cdot \mathrm{e}^{xy}
\end{aligned}$$

Jsou-li ***smíšené derivace*** f_{xy} a f_{yx} v okolí bodu $\mathcal{A}$ ***spojité***, pak jsou si v tomto bodě $\mathcal{A}$ rovny.

Nebo-li: $f_{xy}(\mathcal{A}) = f_{yx}(\mathcal{A})$.

$$\begin{aligned}
f_{xxx} &= y^2 \cdot \mathrm{e}^{xy} \cdot y = y^3 \cdot \mathrm{e}^{xy} \\
f_{xxy} &= 2y \cdot \mathrm{e}^{xy} + y^2 \cdot \mathrm{e}^{xy} \cdot x = (2y + xy^2) \cdot \mathrm{e}^{xy} \\
f_{xyx} &= y \cdot \mathrm{e}^{xy} + (1 + xy) \cdot \mathrm{e}^{xy} \cdot y = (2y + xy^2) \cdot \mathrm{e}^{xy} \\
f_{xyy} &= x \cdot \mathrm{e}^{xy} + (1 + xy) \cdot \mathrm{e}^{xy} \cdot x = (2x + x^2 y) \cdot \mathrm{e}^{xy} \\
f_{yxx} &= y \cdot \mathrm{e}^{xy} + (1 + xy) \cdot \mathrm{e}^{xy} \cdot y = (2y + xy^2) \cdot \mathrm{e}^{xy} \\
f_{yxy} &= x \cdot \mathrm{e}^{xy} + (1 + xy) \cdot \mathrm{e}^{xy} \cdot x = (2x + x^2 y) \cdot \mathrm{e}^{xy} \\
f_{yyx} &= 2x \cdot \mathrm{e}^{xy} + x^2 \cdot \mathrm{e}^{xy} \cdot y = (2x + x^2 y) \cdot \mathrm{e}^{xy} \\
f_{yyy} &= x^2 \cdot \mathrm{e}^{xy} \cdot x = x^3 \cdot \mathrm{e}^{xy}
\end{aligned}$$

Jsou-li ***smíšené derivace*** f_{xxy}, f_{xyx} a f_{yxx} ... ***spojité***, pak jsou si v tomto bodě rovny.

Nebo-li: $f_{xxy}(\mathcal{A}) = f_{xyx}(\mathcal{A}) = f_{yxx}(\mathcal{A})$; $f_{xyy}(\mathcal{A}) = \dots$.

$$\begin{aligned}
f_{xxxx} &= y^3 \cdot \mathrm{e}^{xy} \cdot y = y^4 \cdot \mathrm{e}^{xy} \\
f_{xxxy} &= 3y^2 \cdot \mathrm{e}^{xy} + y^3 \cdot \mathrm{e}^{xy} \cdot x = (3y^2 + xy^3) \cdot \mathrm{e}^{xy} \\
f_{xxyx} &= y^2 \cdot \mathrm{e}^{xy} + (2y + xy^2) \cdot \mathrm{e}^{xy} \cdot y = (3y^2 + xy^3) \cdot \mathrm{e}^{xy} \\
f_{xxyy} &= (2 + 2xy) \cdot \mathrm{e}^{xy} + (2y + xy^2) \cdot \mathrm{e}^{xy} \cdot x = (2 + 4xy + x^2 y^2) \cdot \mathrm{e}^{xy} \\
f_{xyxx} &= y^2 \cdot \mathrm{e}^{xy} + (2y + xy^2) \cdot \mathrm{e}^{xy} \cdot y = (3y^2 + xy^3) \cdot \mathrm{e}^{xy} \\
f_{xyxy} &= (2 + 2xy) \cdot \mathrm{e}^{xy} + (2y + xy^2) \cdot \mathrm{e}^{xy} \cdot x = (2 + 4xy + x^2 y^2) \cdot \mathrm{e}^{xy} \\
f_{xyyx} &= (2 + 2xy) \cdot \mathrm{e}^{xy} + (2x + x^2 y) \cdot \mathrm{e}^{xy} \cdot y = (2 + 4xy + x^2 y^2) \cdot \mathrm{e}^{xy} \\
f_{xyyy} &= x^2 \cdot \mathrm{e}^{xy} + (2x + x^2 y) \cdot \mathrm{e}^{xy} \cdot x = (3x^2 + x^3 y) \cdot \mathrm{e}^{xy} \\
f_{yxxx} &= y^2 \cdot \mathrm{e}^{xy} + (2y + xy^2) \cdot \mathrm{e}^{xy} \cdot y = (3y^2 + xy^3) \cdot \mathrm{e}^{xy} \\
f_{yxxy} &= (2 + 2xy) \cdot \mathrm{e}^{xy} + (2y + xy^2) \cdot \mathrm{e}^{xy} \cdot x = (2 + 4xy + x^2 y^2) \cdot \mathrm{e}^{xy} \\
f_{yxyx} &= (2 + 2xy) \cdot \mathrm{e}^{xy} + (2x + x^2 y) \cdot \mathrm{e}^{xy} \cdot y = (2 + 4xy + x^2 y^2) \cdot \mathrm{e}^{xy} \\
f_{yxyy} &= x^2 \cdot \mathrm{e}^{xy} + (2x + x^2 y) \cdot \mathrm{e}^{xy} \cdot x = (3x^2 + x^3 y) \cdot \mathrm{e}^{xy} \\
f_{yyxx} &= (2 + 2xy) \cdot \mathrm{e}^{xy} + (2x + x^2 y) \cdot \mathrm{e}^{xy} \cdot y = (2 + 4xy + x^2 y^2) \cdot \mathrm{e}^{xy} \\
f_{yyxy} &= x^2 \cdot \mathrm{e}^{xy} + (2x + x^2 y) \cdot \mathrm{e}^{xy} \cdot x = (3x^2 + x^3 y) \cdot \mathrm{e}^{xy} \\
f_{yyyx} &= 3x^2 \cdot \mathrm{e}^{xy} + x^3 \cdot \mathrm{e}^{xy} \cdot y = (3x^2 + x^3 y) \cdot \mathrm{e}^{xy} \\
f_{yyyy} &= x^3 \cdot \mathrm{e}^{xy} \cdot x = x^4 \cdot \mathrm{e}^{xy}
\end{aligned}$$

Jsou-li ***smíšené derivace*** $f_{xxxy}, f_{xxyx}, \dots$

4. $[x \in \mathbb{R};\ y \in \mathbb{R}]$ Napište všechny čtvrté parciální derivace funkce

$$f(x,y) = \cos(xy)$$

$$f_x = -\sin(xy)\cdot y = -y\cdot\sin(xy)$$
$$f_y = -\sin(xy)\cdot x = -x\cdot\sin(xy)$$

$$f_{xx} = -y\cdot\cos(xy)\cdot y = -y^2\cdot\cos(xy)$$
$$f_{xy} = -1\cdot\sin(xy) - y\cdot\cos(xy)\cdot x = -\sin(xy) - xy\cdot\cos(xy)$$
$$f_{yx} = -1\cdot\sin(xy) - x\cdot\cos(xy)\cdot y = -\sin(xy) - xy\cdot\cos(xy)$$
$$f_{yy} = -x\cdot\cos(xy)\cdot x = -x^2\cdot\cos(xy)$$

$$f_{xxx} = -y^2\cdot[-\sin(xy)\cdot y] = y^3\cdot\sin(xy)$$
$$f_{xxy} = -2y\cdot\cos(xy) - y^2\cdot[-\sin(xy)\cdot x] = xy^2\cdot\sin(xy) - 2y\cdot\cos(xy)$$
$$f_{xyx} = -\cos(xy).y - y.\cos(xy) + xy.\sin(xy).y = xy^2\cdot\sin(xy) - 2y\cdot\cos(xy)$$
$$f_{xyy} = -\cos(xy).x - x.\cos(xy) + xy.\sin(xy).x = x^2y\cdot\sin(xy) - 2x\cdot\cos(xy)$$
$$f_{yxx} = -\cos(xy).y - y.\cos(xy) + xy.\sin(xy).y = xy^2\cdot\sin(xy) - 2y\cdot\cos(xy)$$
$$f_{yxy} = -\cos(xy).x - x.\cos(xy) + xy.\sin(xy).x = x^2y\cdot\sin(xy) - 2x\cdot\cos(xy)$$
$$f_{yyx} = -2x\cdot\cos(xy) - x^2\cdot[-\sin(xy)\cdot y] = x^2y\cdot\sin(xy) - 2x\cdot\cos(xy)$$
$$f_{yyy} = -x^2\cdot[-\sin(xy)\cdot x] = x^3\cdot\sin(xy)$$

$$f_{xxxx} = y^3\cdot\cos(xy)\cdot y = y^4\cdot\cos(xy)$$
$$f_{xxxy} = 3y^2\cdot\sin(xy) + y^3\cdot\cos(xy)\cdot x = 3y^2\cdot\sin(xy) + xy^3\cdot\cos(xy)$$
$$f_{xxyx} = y^2.\sin(xy) + xy^2.\cos(xy).y + 2y.\sin(xy).y = 3y^2\cdot\sin(xy) + xy^3\cdot\cos(xy)$$
$$f_{xxyy} = 2xy\cdot\sin(xy) + xy^2\cdot\cos(xy)\cdot x - 2\cdot\cos(xy) - 2y\cdot[-\sin(xy)\cdot x] = 4xy\cdot\sin(xy) + (x^2y^2-2)\cdot\cos(xy)$$
$$f_{xyxx} = y^2.\sin(xy) + xy^2.\cos(xy).y + 2y.\sin(xy).y = 3y^2\cdot\sin(xy) + xy^3\cdot\cos(xy)$$
$$f_{xyxy} = 2xy\cdot\sin(xy) + xy^2\cdot\cos(xy)\cdot x - 2\cdot\cos(xy) - 2y\cdot[-\sin(xy)\cdot x] = 4xy\cdot\sin(xy) + (x^2y^2-2)\cdot\cos(xy)$$
$$f_{xyyx} = 2xy\cdot\sin(xy) + x^2y\cdot\cos(xy)\cdot y - 2\cdot\cos(xy) - 2x\cdot[-\sin(xy)\cdot y] = 4xy\cdot\sin(xy) + (x^2y^2-2)\cdot\cos(xy)$$
$$f_{xyyy} = x^2.\sin(xy) + x^2y.\cos(xy)\cdot x + 2x.\sin(xy).x = 3x^2\cdot\sin(xy) + x^3y\cdot\cos(xy)$$
$$f_{yxxx} = y^2.\sin(xy) + xy^2.\cos(xy).y + 2y.\sin(xy).y = 3y^2\cdot\sin(xy) + xy^3\cdot\cos(xy)$$
$$f_{yxxy} = 2xy\cdot\sin(xy) + xy^2\cdot\cos(xy)\cdot x - 2\cdot\cos(xy) - 2y\cdot[-\sin(xy)\cdot x] = 4xy\cdot\sin(xy) + (x^2y^2-2)\cdot\cos(xy)$$
$$f_{yxyx} = 2xy\cdot\sin(xy) + x^2y\cdot\cos(xy)\cdot y - 2\cdot\cos(xy) - 2x\cdot[-\sin(xy)\cdot y] = 4xy\cdot\sin(xy) + (x^2y^2-2)\cdot\cos(xy)$$
$$f_{yxyy} = x^2.\sin(xy) + x^2y.\cos(xy)\cdot x + 2x.\sin(xy).x = 3x^2\cdot\sin(xy) + x^3y\cdot\cos(xy)$$
$$f_{yyxx} = 2xy\cdot\sin(xy) + x^2y\cdot\cos(xy)\cdot y - 2\cdot\cos(xy) - 2x\cdot[-\sin(xy)\cdot y] = 4xy\cdot\sin(xy) + (x^2y^2-2)\cdot\cos(xy)$$
$$f_{yyxy} = x^2.\sin(xy) + x^2y.\cos(xy)\cdot x + 2x.\sin(xy).x = 3x^2\cdot\sin(xy) + x^3y\cdot\cos(xy)$$
$$f_{yyyx} = 3x^2\cdot\sin(xy) + x^3\cdot\cos(xy)\cdot y = 3x^2\cdot\sin(xy) + x^3y\cdot\cos(xy)$$
$$f_{yyyy} = x^3\cdot\cos(xy)\cdot x = x^4\cdot\cos(xy)$$

5. [$x \in \mathbb{R}$; $y \in \mathbb{R}$] Napište všechny třetí parciální derivace funkce

$$f(x,y) = \sin(x^2 + y^2)$$

$$f_x = \cos(x^2 + y^2) \cdot 2x = 2x \cdot \cos(x^2 + y^2)$$

$$f_y = \cos(x^2 + y^2) \cdot 2y = 2y \cdot \cos(x^2 + y^2)$$

$$f_{xx} = 2.\cos(x^2 + y^2) - 2x.\sin(x^2 + y^2).2x = 2 \cdot \cos(x^2 + y^2) - 4x^2 \cdot \sin(x^2 + y^2)$$

$$f_{xy} = -2x \cdot \sin(x^2 + y^2) \cdot 2y = -4xy \cdot \sin(x^2 + y^2)$$

$$\overline{f_{yx}} = -2y \cdot \sin(x^2 + y^2) \cdot 2x = -4xy \cdot \sin(x^2 + y^2)$$

$$f_{yy} = 2.\cos(x^2 + y^2) - 2y.\sin(x^2 + y^2).2y = 2 \cdot \cos(x^2 + y^2) - 4y^2 \cdot \sin(x^2 + y^2)$$

$$f_{xxx} = -2 \cdot \sin(x^2 + y^2) \cdot 2x - 8x \cdot \sin(x^2 + y^2) - 4x^2 \cdot \cos(x^2 + y^2) \cdot 2x = \\ = -12x \cdot \sin(x^2 + y^2) - 8x^3 \cdot \cos(x^2 + y^2)$$

$$f_{xxy} = -2 \cdot \sin(x^2 + y^2) \cdot 2y - 4x^2 \cdot \cos(x^2 + y^2) \cdot 2y = \\ = -4y \cdot \sin(x^2 + y^2) - 8x^2 y \cdot \cos(x^2 + y^2)$$

$$\overline{f_{xyx}} = -4y \cdot \sin(x^2 + y^2) - 4xy \cdot \cos(x^2 + y^2) \cdot 2x = \\ = -4y \cdot \sin(x^2 + y^2) - 8x^2 y \cdot \cos(x^2 + y^2)$$

$$f_{xyy} = -4x \cdot \sin(x^2 + y^2) - 4xy \cdot \cos(x^2 + y^2) \cdot 2y = \\ = -4x \cdot \sin(x^2 + y^2) - 8xy^2 \cdot \cos(x^2 + y^2)$$

$$\overline{f_{yxx}} = -4y \cdot \sin(x^2 + y^2) - 4xy \cdot \cos(x^2 + y^2) \cdot 2x = \\ = -4y \cdot \sin(x^2 + y^2) - 8x^2 y \cdot \cos(x^2 + y^2)$$

$$f_{yxy} = -4x \cdot \sin(x^2 + y^2) - 4xy \cdot \cos(x^2 + y^2) \cdot 2y = \\ = -4x \cdot \sin(x^2 + y^2) - 8xy^2 \cdot \cos(x^2 + y^2)$$

$$\overline{f_{yyx}} = -2 \cdot \sin(x^2 + y^2) \cdot 2x - 4y^2 \cdot \cos(x^2 + y^2) \cdot 2x = \\ = -4x \cdot \sin(x^2 + y^2) - 8xy^2 \cdot \cos(x^2 + y^2)$$

$$f_{yyy} = -2 \cdot \sin(x^2 + y^2) \cdot 2y - 8y \cdot \sin(x^2 + y^2) - 4y^2 \cdot \cos(x^2 + y^2) \cdot 2y = \\ = -12y \cdot \sin(x^2 + y^2) - 8y^3 \cdot \cos(x^2 + y^2)$$

6. $[x \in \mathbb{R};\ y \in \mathbb{R};\ z \in \mathbb{R}]$ Napište všechny druhé parciální derivace funkce

$$f(x,y,z) = (y^2 + 3z) \cdot \sin(4xz)$$

$$f_x = (y^2 + 3z) \cdot [\cos(4xz) \cdot 4z] = (4y^2z + 12z^2) \cdot \cos(4xz)$$
$$f_y = 2y \cdot \sin(4xz)$$
$$f_z = 3.\sin(4xz) + (y^2 + 3z).[\cos(4xz).4x] = 3\sin(4xz) + (4xy^2 + 12xz) \cdot \cos(4xz)$$

$$f_{xx} = (4y^2z + 12z^2) \cdot [-\sin(4xz) \cdot 4z]$$
$$f_{xy} = 8yz \cdot \cos(4xz)$$
$$f_{xz} = (4y^2 + 24z) \cdot \cos(4xz) + (4y^2z + 12z^2) \cdot [-\sin(4xz) \cdot 4x]$$

$$f_{yx} = 2y \cdot [\cos(4xz) \cdot 4z]$$
$$f_{yy} = 2 \cdot \sin(4xz)$$
$$f_{yz} = 2y \cdot [\cos(4xz) \cdot 4x]$$

$$f_{zx} = 3 \cdot [\cos(xz) \cdot 4z] + (4y^2 + 12z) \cdot \cos(4xz) + (4xy^2 + 12xz) \cdot [-\sin(4xz) \cdot 4z]$$
$$f_{zy} = 8xy \cdot \cos(4xz)$$
$$f_{zz} = 3 \cdot [\cos(4xz) \cdot 4x] + 12x \cdot \cos(4xz) + (4xy^2 + 12xz) \cdot [-\sin(4xz) \cdot 4x]$$

7. $[x \in \mathbb{R};\ y \in \mathbb{R};\ z \in \mathbb{R}]$ Napište všechny druhé parciální derivace funkce

$$f(x,y,z) = (z^2 + 3y) \cdot \cos(4xy)$$

$$f_x = (z^2 + 3y) \cdot [-\sin(4xy) \cdot 4y] = (-4yz^2 - 12y^2) \cdot \sin(4xy)$$
$$f_y = 3\cos(4xy) + (z^2 + 3y).[-\sin(4xy).4x] = (-4xz^2 - 12xy).\sin(4xy) + 3.\cos(4xy)$$
$$f_z = 2z \cdot \cos(4xy)$$

$$f_{xx} = (-4yz^2 - 12y^2) \cdot [\cos(4xy) \cdot 4y]$$
$$f_{xy} = (-4z^2 - 24y) \cdot \sin(4xy) + (-4yz^2 - 12y^2) \cdot [\cos(4xy) \cdot 4x]$$
$$f_{xz} = -8yz \cdot \sin(4xy)$$

$$f_{yx} = (-4z^2 - 12y) \cdot \sin(4xy) + (-4xz^2 - 12xy) \cdot [\cos(4xy).4y] + 3 \cdot [-\sin(4xy).4y]$$
$$f_{yy} = -12x \cdot \sin(4xy) + (-4xz^2 - 12xy) \cdot [\cos(4xy) \cdot 4x] + 3 \cdot [-\sin(4xy) \cdot 4x]$$
$$f_{yz} = -8xz \cdot \sin(4xy)$$

$$f_{zx} = 2z \cdot [-\sin(4xy) \cdot 4y]$$
$$f_{zy} = 2z \cdot [-\sin(4xy) \cdot 4x]$$
$$f_{zz} = 2 \cdot \cos(4xy)$$

8. $[x \in \mathbb{R};\ y \in \mathbb{R};\ z \in \mathbb{R}]$ Napište všechny druhé parciální derivace funkce

$$f(x,y,z) = (x^2 + 3y) \cdot \mathrm{e}^{4yz}$$

$$f_x = 2x \cdot \mathrm{e}^{4yz}$$
$$f_y = 3 \cdot \mathrm{e}^{4yz} + (x^2 + 3y) \cdot [\mathrm{e}^{4yz} \cdot 4z] = (3 + 4x^2 z + 12yz) \cdot \mathrm{e}^{4yz}$$
$$f_z = (x^2 + 3y) \cdot [\mathrm{e}^{4yz} \cdot 4y] = (4x^2 y + 12y^2) \cdot \mathrm{e}^{4yz}$$

$$f_{xx} = 2 \cdot \mathrm{e}^{4yz}$$
$$f_{xy} = 2x \cdot [\mathrm{e}^{4yz} \cdot 4z]$$
$$f_{xz} = 2x \cdot [\mathrm{e}^{4yz} \cdot 4y]$$

$$f_{yx} = 8xz \cdot \mathrm{e}^{4yz}$$
$$f_{yy} = 12z \cdot \mathrm{e}^{4yz} + (3 + 4x^2 z + 12yz) \cdot [\mathrm{e}^{4yz} \cdot 4z]$$
$$f_{yz} = (4x^2 + 12y) \cdot \mathrm{e}^{4yz} + (3 + 4x^2 z + 12yz) \cdot [\mathrm{e}^{4yz} \cdot 4y]$$

$$f_{zx} = 8xy \cdot \mathrm{e}^{4yz}$$
$$f_{zy} = (4x^2 + 24y) \cdot \mathrm{e}^{4yz} + (4x^2 y + 12y^2) \cdot [\mathrm{e}^{4yz} \cdot 4z]$$
$$f_{zz} = (4x^2 y + 12y^2) \cdot [\mathrm{e}^{4yz} \cdot 4y]$$