

$$1. \quad [x \in \mathbb{R}] \quad \underline{\int x \cdot e^{2x} dx} = \left| \begin{array}{ll} u = x & v' = e^{2x} \\ u' = 1 & v = \frac{1}{2}e^{2x} \end{array} \right| = \frac{x}{2}e^{2x} - \frac{1}{2} \int e^{2x} dx =$$

$$= \underline{\underline{\frac{1}{4}e^{2x}(2x - 1) + c}}$$

$$2. \quad [x \in \mathbb{R}] \quad \underline{\int x \cdot \operatorname{arctg} x dx} = \left| \begin{array}{ll} u = \operatorname{arctg} x & v' = x \\ u' = \frac{1}{1+x^2} & v = \frac{x^2}{2} \end{array} \right| =$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{(x^2 + 1) - 1}{1+x^2} dx =$$

$$\underline{\underline{\frac{x^2 + 1}{2} \operatorname{arctg} x - \frac{x}{2} + c}}$$

$$3. \quad [x \in \mathbb{R} \setminus \{0; -1\}] \quad \underline{\int \frac{x^2 - 3x + 2}{x^3 + 2x^2 + x} dx} = \int \left(\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \right) dx =$$

$$\begin{array}{l} x^2 - 3x + 2 = A(x+1)^2 + Bx(x+1) + Cx \\ x = 0 : \quad 2 = A \\ x = -1 : \quad 6 = -C \quad C = -6 \\ x^2 : \quad 1 = A + B \quad B = -1 \end{array}$$

$$= 2 \int \frac{1}{x} dx - \int \underbrace{\frac{1}{x+1}}_{\frac{f'(x)}{f(x)}} dx - 6 \underbrace{\int \frac{1}{(x+1)^2} dx}_{\mathcal{A}(\text{Ize i substitucí})} = \left| \int \frac{1}{(x+1)^2} dx = \frac{-1}{x+1} + c_1 \right|$$

$$\mathcal{A} : \int 1 \cdot \frac{1}{x+1} dx = \left| \begin{array}{ll} u = \frac{1}{x+1} & v' = 1 \\ u' = \frac{-1}{(x+1)^2} & v = x \end{array} \right| = \frac{1}{x+1} \cdot x - \int \frac{-1}{(x+1)^2} \cdot x dx$$

$$\int \frac{1}{x+1} dx = \frac{x}{x+1} + \int \frac{x}{(x+1)^2} dx = \frac{x}{x+1} + \int \frac{(x+1) - 1}{(x+1)^2} dx$$

$$\int \frac{1}{x+1} dx = \frac{x}{x+1} + \int \frac{1}{x+1} dx - \int \frac{1}{(x+1)^2} dx$$

$$\int \frac{1}{(x+1)^2} dx = \frac{x}{x+1} + c_2 = \frac{x}{x+1} \underbrace{- 1 + c_1}_{c_2} = \frac{x - x - 1}{x+1} + c_1 = \frac{-1}{x+1} + c_1$$

$$= 2 \ln |x| - \ln |x+1| - 6 \cdot \left(\frac{-1}{x+1} + c_1 \right) = \ln x^2 - \ln |x+1| + \frac{6}{x+1} \underbrace{- 6c_1}_c =$$

$$\underline{\underline{= \ln \frac{x^2}{|x+1|} + \frac{6}{x+1} + c}}$$

$$\begin{aligned}
 4. \quad [x \in (0; \infty)] \quad \int \frac{\ln x}{x^2} dx &= \left| \begin{array}{ll} u = \ln x & v' = \frac{1}{x^2} \\ u' = \frac{1}{x} & v = -\frac{1}{x} \end{array} \right| = -\frac{\ln x}{x} - \int -\frac{1}{x^2} dx = \\
 &= -\frac{\ln x}{x} + \int x^{-2} dx = -\frac{\ln x}{x} + \frac{x^{-1}}{-1} = \underline{\underline{-\frac{1}{x} \cdot (1 + \ln x) + c}}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad [x \in (-1; 1)] \quad \int \arcsin x dx &= \left| \begin{array}{ll} u = \arcsin x & v' = 1 \\ u' = \frac{1}{\sqrt{1-x^2}} & v = x \end{array} \right| = \\
 &= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx = x \arcsin x + \int \frac{-x}{\sqrt{1-x^2}} dx = \left| \begin{array}{l} w = \sqrt{1-x^2} \\ w^2 = 1-x^2 \\ 2w dw = -2x dx \\ \frac{w}{-x} dw = dx \end{array} \right| = \\
 &= x \arcsin x + \int \frac{-x}{w} \cdot \frac{w}{-x} dw = x \arcsin x + \int dw = x \arcsin x + w + c = \\
 &= \underline{\underline{x \arcsin x + \sqrt{1-x^2} + c}}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad [x \in \mathbb{R}] \quad \int (4x+5) \cdot 2^{3x-1} dx &= \int (4x+5) \cdot e^{(3x-1) \ln 2} dx \\
 &= \left| \begin{array}{ll} u = 4x+5 & v' = e^{(3x-1) \ln 2} \\ u' = 4 & v = \frac{e^{(3x-1) \ln 2}}{3 \ln 2} \end{array} \right| = (4x+5) \cdot \frac{e^{(3x-1) \ln 2}}{3 \ln 2} - 4 \int \frac{e^{(3x-1) \ln 2}}{3 \ln 2} dx = \\
 &= \underline{\underline{\frac{4x+5}{3 \ln 2} \cdot 2^{3x-1} - \frac{4}{(3 \ln 2)^2} \cdot 2^{3x-1} + c}}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad [x \in (1; \infty)] \quad \int x \ln(x-1) dx &= \left| \begin{array}{ll} u = \ln(x-1) & v' = x \\ u' = \frac{1}{x-1} & v = \frac{x^2}{2} \end{array} \right| = \\
 &= \frac{x^2}{2} \ln(x-1) - \frac{1}{2} \int \frac{x^2}{x-1} dx = \frac{x^2}{2} \ln(x-1) - \frac{1}{2} \int \frac{[x(x-1)] + x}{x-1} dx = \\
 &= \frac{x^2}{2} \ln(x-1) - \frac{1}{2} \int x dx - \frac{1}{2} \int \frac{(x-1) + 1}{x-1} dx = \\
 &= \frac{x^2}{2} \ln(x-1) - \frac{1}{2} \cdot \frac{x^2}{2} - \frac{1}{2} \int dx - \frac{1}{2} \int \underbrace{\frac{1}{x-1}}_{\frac{f'(x)}{f(x)}} dx = \\
 &= \frac{x^2}{2} \ln(x-1) - \frac{x^2}{4} - \frac{x}{2} - \frac{1}{2} \ln(x-1) + c = \underline{\underline{-\frac{x^2+2x}{4} + \frac{x^2-1}{2} \ln(x-1) + c}}
 \end{aligned}$$

$$\begin{aligned}
 \text{8. } [x \in \mathbb{R}] \quad \underline{\int x \cdot e^{-x} dx} &= \left| \begin{array}{ll} u = x & v' = e^{-x} \\ u' = 1 & v = -e^{-x} \\ y = -x & dy = -dx \end{array} \right| = -xe^{-x} - \int -e^{-x} dx = \\
 &= \underline{\underline{-\frac{x+1}{e^x} + c}}
 \end{aligned}$$

$$\begin{aligned}
 \text{9. } [x \in \mathbb{R}] \quad \underline{\int x \cos x dx} &= \left| \begin{array}{ll} u = x & v' = \cos x \\ u' = 1 & v = \sin x \end{array} \right| = x \sin x - \int \sin x dx = \\
 &= \underline{\underline{x \sin x + \cos x + c}}
 \end{aligned}$$

$$\begin{aligned}
 \text{10. } [x \in \mathbb{R}] \quad \underline{\int e^x \sin x dx} &= \left| \begin{array}{ll} u_1 = e^x & v'_1 = \sin x \\ u'_1 = e^x & v_1 = -\cos x \end{array} \right| = \\
 = -e^x \cos x - \int -e^x \cos x dx &= -e^x \cos x + \int e^x \cos x dx = \left| \begin{array}{ll} u_2 = e^x & v'_2 = \cos x \\ u'_2 = e^x & v_2 = \sin x \end{array} \right| = \\
 = -e^x \cos x + e^x \sin x - \int e^x \sin x dx &\quad \Downarrow \\
 \underline{\int e^x \sin x dx} &= -e^x \cos x + e^x \sin x - \int e^x \sin x dx \\
 2 \int e^x \sin x dx &= -e^x \cos x + e^x \sin x \\
 \int e^x \sin x dx &= \underline{\underline{\frac{1}{2}e^x \sin x - \frac{1}{2}e^x \cos x + c}}
 \end{aligned}$$

$$\begin{aligned}
 \text{11. } [x \in \mathbb{R}] \quad \underline{\int x^2 \sin x dx} &= \left| \begin{array}{ll} u_1 = x^2 & v'_1 = \sin x \\ u'_1 = 2x & v_1 = -\cos x \end{array} \right| = -x^2 \cos x - \int -2x \cos x dx = \\
 = -x^2 \cos x + \int 2x \cos x dx &= \left| \begin{array}{ll} u_2 = 2x & v'_2 = \cos x \\ u'_2 = 2 & v_2 = \sin x \end{array} \right| = \\
 = -x^2 \cos x + 2x \sin x - \int 2 \sin x dx &= \underline{\underline{-x^2 \cos x + 2x \sin x + 2 \cos x + c}}
 \end{aligned}$$

$$\begin{aligned}
 \text{12. } [x \neq k\pi] \quad \int \frac{x}{\sin^2 x} dx &= \left| \begin{array}{ll} u = x & v' = \frac{1}{\sin^2 x} \\ u' = 1 & v = -\cotg x \end{array} \right| = \\
 &= -x \cotg x - \int \underbrace{-\frac{\cos x}{\sin x}}_{\frac{f'(x)}{f(x)}} dx = \underline{\underline{-x \cotg x + \ln |\sin x| + c}}
 \end{aligned}$$

$$\begin{aligned}
 \text{13. } [x \in (-1; \infty)] \quad \int \ln(x+1) dx &= \left| \begin{array}{ll} u = \ln(x+1) & v' = 1 \\ u' = \frac{1}{x+1} & v = x \end{array} \right| = \\
 &= x \ln(x+1) - \int \frac{x}{x+1} dx = x \ln(x+1) - \int \frac{(x+1)-1}{x+1} dx = \\
 &= x \ln(x+1) - \int \frac{x+1}{x+1} dx - \int \frac{-1}{x+1} dx = x \ln(x+1) - \int dx + \int \underbrace{\frac{1}{x+1}}_{\frac{f'(x)}{f(x)}} dx = \\
 &= \underline{\underline{x \ln(x+1) - x + \ln(x+1) + c}}
 \end{aligned}$$

$$\begin{aligned}
 \text{14. } [x > 0] \quad \int \ln^3 x dx &= \left| \begin{array}{ll} u_1 = \ln^3 x & v'_1 = 1 \\ u'_1 = 3 \ln^2 x \cdot \left(\frac{1}{x}\right) & v_1 = x \end{array} \right| = x \ln^3 x - \int 3 \ln^2 x dx = \\
 &= \left| \begin{array}{ll} u_2 = \ln^2 x & v'_2 = 1 \\ u'_2 = 2 \ln x \cdot \left(\frac{1}{x}\right) & v_2 = x \end{array} \right| = x \ln^3 x - 3 \left(x \ln^2 x - \int 2 \ln x dx \right) = \\
 &= \left| \begin{array}{ll} u_3 = \ln x & v'_3 = 1 \\ u'_3 = \frac{1}{x} & v_3 = x \end{array} \right| = x \ln^3 x - 3x \ln^2 x + 6 \left(x \ln x - \int dx \right) = \\
 &= \underline{\underline{x \ln^3 x - 3x \ln^2 x + 6x \ln x - 6x + c}}
 \end{aligned}$$