1. $[x \in \mathbb{R} \setminus \{0\}]$ $\underline{\displaystyle\int \frac{3x^4 + 5x^3 - 6x \cdot \sqrt[3]{x} + 4}{x}\, \mathrm{d}x} = \displaystyle\int \left(3x^3 + 5x^2 - 6x^{\frac{1}{3}} + \frac{4}{x}\right) \mathrm{d}x =$

$$= 3\,\frac{x^{3+1}}{4} + 5\,\frac{x^{2+1}}{3} - 6\,\frac{x^{\frac{1}{3}+1}}{\frac{4}{3}} + 4\ln|x| + c = \underline{\underline{\frac{3}{4}x^4 + \frac{5}{3}x^3 - \frac{9}{2}\cdot\sqrt[3]{x^4} + \ln x^4 + c}}$$

2. $[x \in \mathbb{R} \setminus \{0; -1\}]$ $\underline{\displaystyle\int \frac{x^2 - 3x + 2}{x^3 + 2x^2 + x}\, \mathrm{d}x} = \displaystyle\int \left(\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right) \mathrm{d}x =$

$$x^2 - 3x + 2 = A\,(x+1)^2 + B\,x(x+1) + C\,x$$

$$\begin{array}{rll} x = 0: & 2 = A & \\ x = -1: & 6 = -C & C = -6 \\ x^2: & 1 = A + B & B = -1 \end{array}$$

$$= 2\int \frac{1}{x}\,\mathrm{d}x - \int \underbrace{\frac{1}{x+1}}_{\frac{f'(x)}{f(x)}}\,\mathrm{d}x - 6\int \underbrace{\frac{1}{(x+1)^2}\,\mathrm{d}x}_{\text{substituce}} = \left| \begin{array}{ll} y = x + 1 & \\ y' = 1 & \\ \frac{\mathrm{d}y}{\mathrm{d}x} = 1 & \frac{1}{y^2} = y^{-2} \\ \mathrm{d}y = \mathrm{d}x & \end{array} \right| =$$

$$= 2\ln|x| - \ln|x+1| - 6\int y^{-2}\,\mathrm{d}y = \ln\frac{x^2}{|x+1|} - 6\cdot\frac{y^{-1}}{-1} + c = \underline{\underline{\ln\frac{x^2}{|x+1|} + \frac{6}{x+1} + c}}$$

3. $[x \in \mathbb{R} \setminus \{-1\}]$ $\underline{\displaystyle\int \frac{x-1}{x+1}\,\mathrm{d}x} = \displaystyle\int \frac{(x+1)-2}{x+1}\,\mathrm{d}x = \int \left(\frac{x+1}{x+1} - \frac{2}{x+1}\right) \mathrm{d}x =$

$$= \int 1\,\mathrm{d}x - 2\int \frac{1}{x+1}\,\mathrm{d}x = \left|\int \frac{f'(x)}{f(x)}\,\mathrm{d}x\right| = x - 2\ln|x+1| + c = \underline{\underline{x - \ln(x+1)^2 + c}}$$

4. $[x \in \mathbb{R}]$ $\underline{\displaystyle\int \frac{x^2-1}{x^2+1}\,\mathrm{d}x} = \displaystyle\int \frac{(x^2+1)-2}{x^2+1}\,\mathrm{d}x = \int \left(\frac{x^2+1}{x^2+1} - \frac{2}{x^2+1}\right) \mathrm{d}x =$

$$= \int 1\,\mathrm{d}x - \int \frac{2}{x^2+1}\,\mathrm{d}x = \underline{\underline{x - 2\operatorname{arctg} x + c}}$$

5. $[x \in \mathbb{R}]$ $\underline{\displaystyle\int \frac{x^4}{x^2+1}\,\mathrm{d}x} = \displaystyle\int \frac{(x^4+x^2)-x^2}{x^2+1}\,\mathrm{d}x = \int \left[\frac{x^2(x^2+1)}{x^2+1} - \frac{(x^2+1)-1}{x^2+1}\right] \mathrm{d}x =$

$$= \int x^2\,\mathrm{d}x - \int 1\,\mathrm{d}x - \int \frac{-1}{x^2+1}\,\mathrm{d}x = \underline{\underline{\frac{1}{3}x^3 - x + \operatorname{arctg} x + c}}$$

6. $[x \in \mathbb{R} \setminus \{1\}]$ $\quad \underline{\int \frac{x}{x^3-1}\,\mathrm{d}x} = \int \frac{x}{(x-1)(x^2+x+1)}\,\mathrm{d}x =$

$$= \int \left(\frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \right) \mathrm{d}x =$$

$$x = A(x^2+x+1) + (Bx+C)(x-1)$$

$$\begin{array}{lll} x = 1: & 1 = 3A & A = \frac{1}{3} \\ x^2: & 0 = A + B & B = -\frac{1}{3} \\ x^0: & 0 = A - C & C = \frac{1}{3} \end{array}$$

$$= \int \left(\frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{3}x + \frac{1}{3}}{x^2+x+1} \right) \mathrm{d}x = \frac{1}{3} \int \underbrace{\frac{1}{x-1}}_{\frac{f'(x)}{f(x)}} \mathrm{d}x - \frac{1}{3} \int \frac{x-1}{x^2+x+1}\,\mathrm{d}x =$$

$$= \frac{1}{3} \ln|x-1| - \frac{1}{3} \cdot \frac{1}{2} \int \frac{(2x+1)-2}{x^2+x+1}\,\mathrm{d}x =$$

$$= \frac{1}{3} \ln|x-1| - \frac{1}{6} \int \underbrace{\frac{2x+1}{x^2+x+1}}_{\frac{f'(x)}{f(x)}} \mathrm{d}x + \frac{1}{3} \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}\,\mathrm{d}x = \left| \begin{array}{r} x + \frac{1}{2} = \sqrt{\frac{3}{4}} \cdot u \\ \mathrm{d}x = \frac{\sqrt{3}}{2}\,\mathrm{d}u \\ \frac{2x+1}{\sqrt{3}} = u \end{array} \right| =$$

$$= \frac{1}{6} \left(2 \ln|x-1| - \ln|\underbrace{x^2+x+1}_{\text{kompl. k.}}| \right) + \frac{1}{3} \int \frac{1}{\frac{3}{4}u^2 + \frac{3}{4}} \cdot \frac{\sqrt{3}}{2}\,\mathrm{d}u =$$

$$= \frac{1}{6} \ln \frac{(x-1)^2}{x^2+x+1} + \frac{1}{3} \cdot \frac{1}{\frac{3}{4}} \cdot \frac{\sqrt{3}}{2} \int \frac{1}{u^2+1}\,\mathrm{d}x = \frac{1}{6} \ln \frac{(x-1)^2}{x^2+x+1} + \frac{2\sqrt{3}}{9} \operatorname{arctg} u + c =$$

$$= \underline{\underline{\frac{1}{6} \ln \frac{(x-1)^2}{x^2+x+1} + \frac{2\sqrt{3}}{9} \operatorname{arctg} \frac{2x+1}{\sqrt{3}} + c}}$$

7. $[x \in (-2\,;\,2)]$ $\quad \underline{\int \frac{x}{\sqrt{4-x^2}}\,\mathrm{d}x} = \left| \begin{array}{r} 4 - x^2 = s \\ -2x\,\mathrm{d}x = \mathrm{d}s \\ \mathrm{d}x = \frac{-1}{2x}\,\mathrm{d}s \end{array} \right| = \int \frac{x}{\sqrt{s}} \cdot \frac{-1}{2x}\,\mathrm{d}s =$

$$= -\frac{1}{2} \int s^{-\frac{1}{2}}\,\mathrm{d}s = -\frac{1}{2} \cdot \frac{s^{\frac{1}{2}}}{\frac{1}{2}} + c = -\sqrt{s} + c = \underline{\underline{-\sqrt{4-x^2} + c}}$$

8. $[x \in (0\,;\,\infty)]$ $\underline{\displaystyle\int \frac{\ln^2 x}{x}\,\mathrm{d}x} = \left|\begin{array}{c} \ln x = v \\ \frac{1}{x}\,\mathrm{d}x = \mathrm{d}v \\ \mathrm{d}x = x\,\mathrm{d}v \end{array}\right| = \displaystyle\int \frac{v^2}{x}\cdot x\,\mathrm{d}v = \int v^2\,\mathrm{d}v = \frac{1}{3}v^3 + c =$

$$= \underline{\underline{\frac{1}{3}\ln^3 x + c}}$$

9. $[x \in \mathbb{R}]$ $\underline{\displaystyle\int \frac{\sqrt[3]{\operatorname{arctg} x}}{1+x^2}\,\mathrm{d}x} = \left|\begin{array}{c} \operatorname{arctg} x = v \\ \frac{1}{1+x^2}\,\mathrm{d}x = \mathrm{d}v \\ \mathrm{d}x = (1+x^2)\,\mathrm{d}v \end{array}\right| = \displaystyle\int \frac{\sqrt[3]{v}}{1+x^2}\cdot(1+x^2)\,\mathrm{d}v =$

$$= \int \sqrt[3]{v}\,\mathrm{d}v = \int v^{\frac{1}{3}}\,\mathrm{d}v = \frac{v^{\frac{4}{3}}}{\frac{4}{3}} + c = \underline{\underline{\frac{3}{4}\cdot\sqrt[3]{\operatorname{arctg}^4 x} + c}}$$

10. $[x \in (0\,;\,\infty)]$ $\underline{\displaystyle\int \frac{\cos(\ln x)}{x}\,\mathrm{d}x} = \left|\begin{array}{c} \ln x = w \\ \frac{1}{x}\,\mathrm{d}x = \mathrm{d}w \\ \mathrm{d}x = x\,\mathrm{d}w \end{array}\right| = \displaystyle\int \frac{\cos w}{x}\cdot x\,\mathrm{d}w = \int \cos w\,\mathrm{d}w =$

$$= \sin w + c = \underline{\underline{\sin(\ln x) + c}}$$

11. $\left[x \neq (2k+1)\,\frac{\pi}{2}\right]$ $\underline{\displaystyle\int \frac{\mathrm{e}^{\operatorname{tg} x}}{\cos^2 x}\,\mathrm{d}x} = \left|\begin{array}{c} \operatorname{tg} x = y \\ \frac{1}{\cos^2 x}\,\mathrm{d}x = \mathrm{d}y \\ \mathrm{d}x = \cos^2 x\,\mathrm{d}y \end{array}\right| = \displaystyle\int \frac{\mathrm{e}^y}{\cos^2 x}\cdot\cos^2 x\,\mathrm{d}y =$

$$= \int \mathrm{e}^y\,\mathrm{d}y = \mathrm{e}^y + c = \underline{\underline{\mathrm{e}^{\operatorname{tg} x} + c}}$$

12. $[x \in (0\,;\,\infty)]$ $\underline{\displaystyle\int \frac{\ln^2 x + 1}{x}\,\mathrm{d}x} = \left|\begin{array}{c} \ln x = v \\ \frac{1}{x}\,\mathrm{d}x = \mathrm{d}v \\ \mathrm{d}x = x\,\mathrm{d}v \end{array}\right| = \displaystyle\int \frac{v^2+1}{x}\cdot x\,\mathrm{d}v =$

$$= \int (v^2+1)\,\mathrm{d}v = \frac{1}{3}v^3 + v + c = \underline{\underline{\frac{1}{3}\ln^3 x + \ln x + c}}$$

13. $[x \in \mathbb{R}]$

$$\underline{\int \frac{2x-1}{\sqrt{1-x+x^2}}\,\mathrm{d}x} = \left| \begin{array}{r} 1-x+x^2=u \\ (-1+2x)\,\mathrm{d}x = \mathrm{d}u \\ \mathrm{d}x = \frac{1}{2x-1}\,\mathrm{d}u \end{array} \right| =$$

$$= \int \frac{2x-1}{\sqrt{u}} \cdot \frac{1}{2x-1}\,\mathrm{d}u = \int u^{-\frac{1}{2}}\,\mathrm{d}u = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} = \underline{\underline{2\sqrt{1-x+x^2}+c}}$$

14. $[x \in \mathbb{R}]$

$$\underline{\int (x^2+1)^3 \cdot 2x\,\mathrm{d}x} = \left| \begin{array}{r} x^2+1=t \\ 2x\,\mathrm{d}x = \mathrm{d}t \\ \mathrm{d}x = \frac{1}{2x}\,\mathrm{d}t \end{array} \right| = \int t^3 \cdot 2x \cdot \frac{1}{2x}\,\mathrm{d}t = \int t^3\,\mathrm{d}t =$$

$$= \frac{t^4}{4} + c = \underline{\underline{\frac{1}{4}(x^2+1)^4 + c}}$$

15. $\left[x \in \left(0\,;\frac{1}{\mathrm{e}}\right) \cup \left(\frac{1}{\mathrm{e}}\,;\infty\right)\right]$

$$\underline{\int \frac{1}{x\cdot(1+\ln x)}\,\mathrm{d}x} = \left| \begin{array}{r} 1+\ln x = z \\ \frac{1}{x}\,\mathrm{d}x = \mathrm{d}z \\ \mathrm{d}x = x\,\mathrm{d}z \end{array} \right| =$$

$$= \int \frac{1}{x\cdot z}\cdot x\,\mathrm{d}z = \int \frac{1}{z}\,\mathrm{d}z = \ln|z| + c = \underline{\underline{\ln|1+\ln x| + c}}$$

16. $[x \in (1\,;3)]$

$$\underline{\int \frac{1}{\sqrt{4x-3-x^2}}\,\mathrm{d}x} = \int \frac{1}{\sqrt{-3-(x^2-4x)}}\,\mathrm{d}x =$$

$$= \int \frac{1}{\sqrt{-3-(x^2-4x+4-4)}}\,\mathrm{d}x = \int \frac{1}{\sqrt{-3-(x^2-4x+4)+4}}\,\mathrm{d}x =$$

$$= \int \frac{1}{\sqrt{1-(x-2)^2}}\,\mathrm{d}x = \left| \begin{array}{r} x-2=y \\ \mathrm{d}x = \mathrm{d}y \end{array} \right| = \int \frac{1}{\sqrt{1-y^2}}\,\mathrm{d}y = \arcsin y + c =$$

$$= \underline{\underline{\arcsin(x-2) + c}}$$

17. $[x \in \mathbb{R} \setminus \{\pm 2\}]$

$$\underline{\int \frac{2x^2-8}{16-x^4}\,\mathrm{d}x} = \int \frac{2(x^2-4)}{(4-x^2)(4+x^2)}\,\mathrm{d}x = -2\int \frac{1}{4+x^2}\,\mathrm{d}x =$$

$$= -2\int \frac{1}{4\left(1+\frac{x^2}{4}\right)}\,\mathrm{d}x = -\frac{1}{2}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}\,\mathrm{d}x = \left| \begin{array}{r} \frac{x}{2} = u \\ x = 2u \\ \mathrm{d}x = 2\,\mathrm{d}u \end{array} \right| =$$

$$= -\frac{1}{2}\int \frac{1}{1+u^2}\cdot 2\,\mathrm{d}u = -\operatorname{arctg} u + c = \underline{\underline{-\operatorname{arctg}\frac{x}{2} + c}}$$

18. $[x \in (0\,;\,1) \cup (1\,;\,\infty)]$ $\quad \underline{\displaystyle\int \frac{\sqrt{x}}{\sqrt[3]{x^2}-\sqrt[4]{x}}\,\mathrm{d}x} = \displaystyle\int \frac{x^{\frac{1}{2}}}{x^{\frac{2}{3}}-x^{\frac{1}{4}}}\,\mathrm{d}x =$

$$= \left| \frac{1}{2};\frac{2}{3};\frac{1}{4} \Rightarrow \text{společný jmenovatel je } 12 \Longrightarrow \begin{array}{r} x = z^{12} \\ \mathrm{d}x = 12z^{11}\,\mathrm{d}z \\ \sqrt[12]{x} = z \end{array} \right| =$$

$$= \int \frac{(z^{12})^{\frac{1}{2}}}{(z^{12})^{\frac{2}{3}}-(z^{12})^{\frac{1}{4}}}\,12z^{11}\,\mathrm{d}z = 12\int \frac{z^6}{z^8-z^3}\,z^{11}\,\mathrm{d}z = 12\int \frac{z^{17}}{z^3(z^5-1)}\,\mathrm{d}z =$$

$$= 12\int \frac{z^{14}}{z^5-1}\,\mathrm{d}z = 12\int \frac{z^{10}}{z^5-1}\,z^4\,\mathrm{d}z = \left| \begin{array}{r} z^5-1 = t \\ z^5 = t+1 \\ 5z^4\,\mathrm{d}z = \mathrm{d}t \\ z^4\,\mathrm{d}z = \frac{1}{5}\,\mathrm{d}t \\ z^{10} = (z^5)^2 \end{array} \right| = 12\int \frac{(t+1)^2}{t}\cdot\frac{1}{5}\,\mathrm{d}t =$$

$$= \frac{12}{5}\int \frac{t^2+2t+1}{t}\,\mathrm{d}t = \frac{12}{5}\int \left(t+2+\frac{1}{t}\right)\mathrm{d}t = \frac{12}{5}\left(\frac{t^2}{2}+2t+\ln|t|\right)+c =$$

$$= \frac{12}{10}\left[(z^5-1)^2+4(z^5-1)+2\ln|(z^5-1)|\right]+c =$$

$$= \underline{\underline{\frac{12}{10}\left[\left(\sqrt[12]{x^5}-1\right)^2+4\left(\sqrt[12]{x^5}-1\right)+\ln\left(\sqrt[12]{x^5}-1\right)^2\right]+c}}$$

19. $[x \in (-2\,;\,2)]$ $\quad \underline{\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,\mathrm{d}x} = \left| \begin{array}{c} x = 2\sin y \\ \mathrm{d}x = 2\cos y\,\mathrm{d}y \\ \frac{x}{2} = \sin y \\ \arcsin\frac{x}{2} = y \end{array} \right| =$

$$= \int \frac{(2\sin y)^2}{\sqrt{4-(2\sin y)^2}}\,2\cos y\,\mathrm{d}y = \int \frac{4\sin^2 y}{\sqrt{4-4\sin^2 y}}\,2\cos y\,\mathrm{d}y =$$

$$= \int \frac{8\sin^2 y\cdot\cos y}{\sqrt{4}\cdot\sqrt{1-\sin^2 y}}\,\mathrm{d}y = \int \frac{8\sin^2 y\cdot\cos y}{2\sqrt{\cos^2 y}}\,\mathrm{d}y = 4\int \frac{\sin^2 y\cdot\cos y}{|\cos y|}\,\mathrm{d}y =$$

$$= \left| \begin{array}{c} \arcsin\frac{x}{2} = y \\ H\left(\arcsin\frac{x}{2}\right) = \left\langle -\frac{\pi}{2}\,;\,\frac{\pi}{2}\right\rangle \\ \left(-\frac{\pi}{2}\,;\,\frac{\pi}{2}\right) = D(y) \\ \cos y > 0 \end{array} \right| = 4\int \frac{\sin^2 y\cdot\cos y}{\cos y}\,\mathrm{d}y = 4\int \sin^2 y\,\mathrm{d}y =$$

$$= 4\int \frac{1-\cos 2y}{2}\,\mathrm{d}y = 2\int \mathrm{d}y - \int \cos 2y\cdot 2\,\mathrm{d}y = \left| \begin{array}{c} 2y = z \\ 2\,\mathrm{d}y = \mathrm{d}z \end{array} \right| =$$

$$= 2y - \int \cos z\,\mathrm{d}z = 2\arcsin\frac{x}{2} - \sin z + c = 2\arcsin\frac{x}{2} - \sin 2y + c =$$

$$= 2\arcsin\frac{x}{2} - 2\sin y\cdot\cos y + c = 2\arcsin\frac{x}{2} - x\sqrt{1-\sin^2 y} + c =$$

$$= 2\arcsin\frac{x}{2} - x\sqrt{1-\left(\frac{x}{2}\right)^2} + c = 2\arcsin\frac{x}{2} - x\sqrt{\frac{4-x^2}{4}} + c =$$

$$= \underline{\underline{2\arcsin\frac{x}{2} - \frac{x}{2}\sqrt{4-x^2} + c}}$$