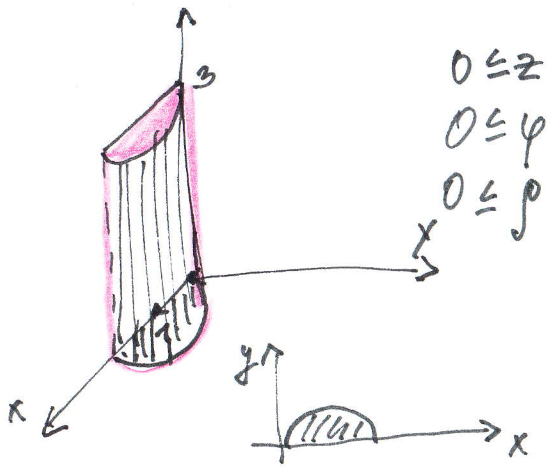
1)



$0 \le z \le 3$

$0 \le \varphi \le \frac{\pi}{2}$

$0 \le \rho \le 2\cos\varphi$

$z = z$

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

$J = \rho$

$\rho^2 - 2\rho\cos\varphi = 0$

$\rho = 2\cos\varphi$

$$\int_0^3\left(\int_0^{\frac{\pi}{2}}\left(\int_0^{2\cos\varphi} z\,\rho\cdot\rho\,d\rho\right)d\varphi\right)dz = \int_0^3\left(\int_0^{\frac{\pi}{2}} z\left[\frac{\rho^3}{3}\right]_0^{2\cos\varphi}d\varphi\right)dz$$

$$= \frac{1}{3}\int_0^3 z\left(\int_0^{\frac{\pi}{2}} 8\cos^3\varphi\,d\varphi\right)dz = \frac{8}{3}\int_0^3 z\cdot\frac{2}{3}\,dz =$$

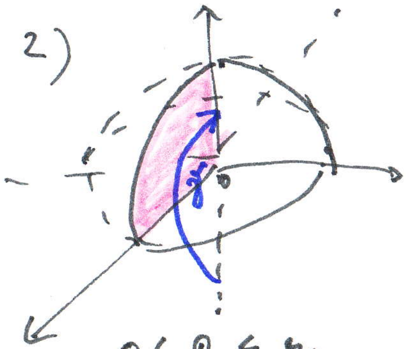
$$= \frac{16}{9}\left[\frac{z^2}{2}\right]_0^3 = \frac{16}{\cancel{9}}\cdot\frac{1}{2}\cdot\cancel{9} = \underline{\underline{8}}$$

$$\int_0^{\frac{\pi}{2}}\cos^3\varphi\,d\varphi = \left|\begin{array}{l}\sin\varphi = t\\ \cos\varphi\,d\varphi = dt\\ 0 \to 0\\ \frac{\pi}{2} \to 1\end{array}\right| = \int_0^1(1-t^2)\,dt = \left[t-\frac{t^3}{3}\right]_0^1 =$$

$$\underbrace{\cos^2\varphi}_{1-t^2}\cdot\underbrace{\cos\varphi\,d\varphi}_{dt}$$

$$= \frac{2}{3}$$

2)



$0 \le \rho \le r$

$0 \le \varphi \le 2\pi$

$0 \le \gamma \le \frac{\pi}{2}$

$$x = \rho \cos\varphi \cos\gamma$$
$$y = \rho \sin\varphi \cos\gamma$$
$$z = \rho \sin\gamma$$

$$J = \rho^2 \cos\gamma$$

$$\int_0^r \left(\int_0^{2\pi} \left(\int_0^{\frac{\pi}{2}} \rho^2 \cos^2\varphi \cos^2\gamma \cdot \rho \sin\gamma \cdot \rho^2 \cos\gamma \, d\gamma \right) d\varphi \right) d\rho =$$

$$= \int_0^r \left(\int_0^{2\pi} \rho^5 \cos^2\varphi \left(\int_0^{2\pi} \cos^3\gamma \sin\gamma \, d\gamma \right) d\varphi \right) d\rho =$$

$$\int_0^{2\pi} \cos^3\gamma \sin\gamma \, d\gamma \quad \left| \begin{array}{l} \cos\gamma = t \\ -\sin\gamma \, d\gamma = dt \\ 0 \to 1 \\ \frac{\pi}{2} \to 0 \end{array} \right| = \int_0^1 t^3 \, dt = \left[\frac{t^4}{4} \right]_0^1 = \frac{1}{4}$$

$$= \frac{1}{4} \int_0^r \rho^5 \left(\int_0^{2\pi} \underbrace{\cos^2\varphi}_{\frac{1+\cos 2\varphi}{2}} \, d\varphi \right) d\rho = \frac{1}{4} \int_0^r \rho^5 \cdot \frac{1}{2} \left[\varphi + \sin 2\varphi \cdot \frac{1}{2} \right]_0^{2\pi} d\rho =$$

$$= \frac{1}{8} \int_0^r \rho^5 \cdot 2\pi \, d\rho = \frac{\pi}{4} \left[\frac{\rho^6}{6} \right]_0^r = \frac{\pi}{24} r^6$$

3) $0 \le \rho \le r$ $\qquad \sigma(\rho, \varphi, \vartheta) = k \cdot \rho$

$0 \le \varphi \le 2\pi$

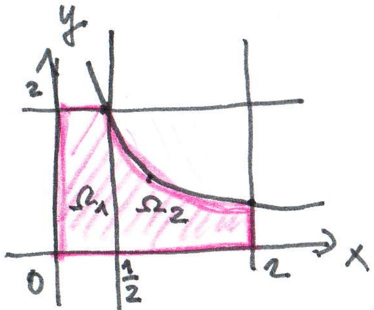
$-\frac{\pi}{2} \le \vartheta \le \frac{\pi}{2}$

$$\int_0^r \left(\int_0^{2\pi} \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} k \cdot \rho \cdot \rho^2 \cos\vartheta \, d\vartheta \right) d\varphi \right) d\rho =$$

$$= k \int_0^r \rho^3 \left(\int_0^{2\pi} \underbrace{\left[\sin\vartheta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}_{1-(-1)} d\varphi \right) d\rho = 2k \int_0^r \rho^3 \left[\varphi\right]_0^{2\pi} d\rho =$$

$$= 2k \cdot 2\pi \left[\frac{\rho^4}{4} \right]_0^r = \cancel{4} k\pi \cdot \frac{r^4}{\cancel{4}} = \underline{k\pi r^4}$$

4(3))



$0 \le z \le 3$	$0 \le z \le 3$
$0 \le x \le \frac{1}{2}$	$\frac{1}{2} \le x \le 2$
$0 \le y \le 2$	$0 \le y \le \frac{1}{x}$

$$\Omega_1 : \int_0^3 \left(\int_0^{\frac{1}{2}} \left(\int_0^2 dy \right) dx \right) dz = [z]_0^3 [x]_0^{\frac{1}{2}} [y]_0^2 = 3 \cdot \frac{1}{2} \cdot 2 = \underline{\underline{3}}$$

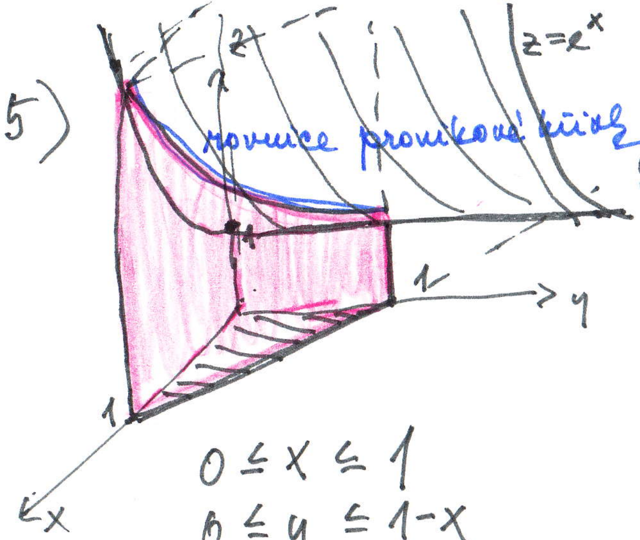
$$\Omega_2 : \int_0^3 \left(\int_{\frac{1}{2}}^2 \left(\int_0^{\frac{1}{x}} dy \right) dx \right) dz = \int_0^3 \left(\int_{\frac{1}{2}}^2 [y]_0^{\frac{1}{x}} dx \right) dz =$$

$$= \int_0^3 \left(\int_{\frac{1}{2}}^2 \frac{1}{x}\, dx \right) dz = \int_0^3 [\ln x]_{\frac{1}{2}}^2 \, dz = \left(\ln 2 - \ln \frac{1}{2} \right) [z]_0^3 =$$

$$= 3 \left(\ln 2 - (-\ln 2) \right) = \underline{\underline{6 \ln 2}}$$

$$\Omega = \Omega_1 \cup \Omega_2 = \underline{\underline{3 + 6 \ln 2}}$$

5)



rovnice průnikové křivky : $\left.\begin{aligned} x &= t \\ y &= 1-t \\ z &= e^t \end{aligned}\right\} t \in \langle 0, 1 \rangle$ ... odpověď na dotaz

$0 \le x \le 1$

$0 \le y \le 1-x$

$0 \le z \le e^x$

$\rho(x,y,z) = k$

$$\int_0^1 \left(\int_0^{1-x} \left(\int_0^{e^x} x \cdot k \, dz \right) dy \right) dx =$$

$$= k \int_0^1 x \left(\int_0^{1-x} [z]_0^{e^x} \, dy \right) dx =$$

$$= k \int_0^1 x \left(\int_0^{1-x} e^x \, dy \right) dx = k \int_0^1 x e^x [y]_0^{1-x} \, dx =$$

$$= k \int_0^1 x e^x (1-x) \, dx = k \int_0^1 e^x (x - x^2) \, dx =$$

$$\left| \begin{matrix} u = x - x^2 & v' = e^x \\ u' = 1-2x & v = e^x \end{matrix} \right| = k \left(\underbrace{\left[(x-x^2) e^x \right]_0^1}_{0} - \int_0^1 e^x (1-2x) \, dx \right) =$$

$$\left| \begin{matrix} u = 1-2x & v' = e^x \\ u' = -2 & v = e^x \end{matrix} \right| = k(-1) \cdot \left[\left[(1-2x) e^x \right]_0^1 - \int_0^1 (-2) e^x \, dx \right] =$$

$$= -k \left(\left(e(1-2) - (1) \cdot 1 + 2 \left[e^x \right]_0^1 \right) \right) =$$

$$= -k \left(-e - 1 + 2(e-1) \right) = -k(e-3) = \underline{\underline{k(3-e)}}$$