Economic Applications in Teaching Mathematics at the Economic College

Ekonomické aplikace ve výuce matematiky na vysokých školách ekonomických

Pavel Kovařík, Rudolf Schwarz

Abstract

We can find mathematical applications in overt or covert form in almost all branches of human activities. If we consider only education sector we can find it in all technical, economic but also other disciplines. In this paper we have introduced some customary and not so frequently used methods and ways of their usage at the particular college. Due to the orientation of the school the economic applications are preferred.

1. Úvod

At Karel Englis College, Inc. mathematics is taught in first two terms with the 2/2 hours time–donation with the following content:

1. Basics of linear algebra; sequences and their applications.
2. Infinitesimal (differential and integral) calculus.

Economic and managerial application incorporated into these topics follows.

2. Matrices

The concept of matrices and the operations with them can be used very well if matrices are interpreted as ordered or sold amount of products, prices of products, labor costs, amount of desirable raw materials or their prices. Then the task can be formulated as follows:
Let's have matrices

> $\boldsymbol{A}$ contains time necessary for manufacturing, wrapping and expedition of products where rows point to individual products (e.g. tables and chairs) and columns point to appropriate professions (cabinetmaker, painter, warehouseman)

$$\boldsymbol{A} = \begin{array}{cc} \begin{bmatrix} 6 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix} & \begin{array}{l} \text{table} \\ \text{chair} \end{array} \\ \begin{array}{ccc} \text{cabinetmaker} & \text{painter} & \text{warehouseman} \end{array} & \end{array}$$

> $\boldsymbol{B}$ contains labor costs per unit corresponding to appropriate professions, $\boldsymbol{B} = [80 \quad 50 \quad 40]$.

Find the product $\boldsymbol{A} \cdot \boldsymbol{B}^T$ and interpret the result economically.

$$\boldsymbol{A} \cdot \boldsymbol{B}^T = \begin{bmatrix} 6 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 80 \\ 50 \\ 40 \end{bmatrix} = \begin{bmatrix} 760 \\ 380 \end{bmatrix}$$

Elements of the resulting matrix enable us to see labor cost for workers involved in the production of goods under consideration.

3. Systems of linear equations

Systems of linear equations use matrices so their applications track the same line. To find raw material cost per unit necessary for various products belongs to basic problems. The ratios between individual components and their available cost must be known.

This principle can be expressed by a matrix equation

$$\boldsymbol{C} \cdot \boldsymbol{X} = \boldsymbol{N}$$

where $\boldsymbol{C}$ contains ratios between individual components, $\boldsymbol{N}$ contains allowed costs and $\boldsymbol{X}$ contains row material costs per unit which are to be found. The formula reflects the fact that prices per unit times quantity gives total price.

4. Sequences (banking products)

From the application point of view arithmetic and geometrical sequences are emphasized. Mainly their sums are important. It is considered that the amount of money in the bank is once changed $(1+i)$–times per each year, where i is the interest rate p.a. Then, after n years, we have a geometrical sequence with the quotient $q = (1+i)$. Using this principle we come to a basic formula $K = V \cdot (1+i)^n$.

When the interest period is shorter than one year — let's say m–times a year — we get the formula

$$K = V \cdot \left(1 + \frac{i}{m}\right)^{m \cdot n + z}$$

where z represents the remaining interest periods over whole years.

Introducing the concept of discount $v = \frac{1}{1+i}$ we can use the partial sum of both arithmetic and geometrical sequence with the quotient v for savings, pensions and loans. Because the problem is often divided to partial steps (e.g. under– or over–year savings) the principle of sequence sum is used in combination. E.g. for postponed pension (the recipient waits for the first payment certain time period and the payment itself is composed from under– and over–year parts) we obtain the following formula by combining these parts

$$D_0 = \frac{a}{i} \cdot [m + 0,5 \cdot (m+1) \cdot i] \cdot [1 - v^n] \cdot v^k,$$

Where n represents the time period for taking the pension and k is the time period of waiting for the beginning.

Especially so called eternal pension (without delay) enables to use limit of function. The function is represented by the formula for postponed pension where $k = 0$ and limit is defined for $n \to \infty$. Since $0 < v < 1$, we get $v^0 = 1$. So

$$D_v = \lim_{n \to \infty} \frac{a}{i} \cdot [m + 0,5 \cdot (m+1) \cdot i] \cdot [1 - v^n] \cdot v^0 = \frac{a}{i} \cdot [m + 0,5 \cdot (m+1) \cdot i]$$

Obviously it is a pension where the payment is equal to interest.

We usually require redemption plan to be built for the case of loans. I.e. we ask for calculation of the precise value of installment so the debt will be repaid in the end of agreed time. The balance in the end of every year must be also involved.

5. Derivatives (marginal quantities, flow, intensity)

The known technical (physical) application $s'(t) = \frac{\mathrm{d}s}{\mathrm{d}t}$ represents an immediate velocity of motion at any given time. The similar economic interpretation of a derivative describes the velocity of a change of some economic quantity. Such a new quantity is usually called marginal quantity. E.g. $TR(Q)$ represents the total income, Q is a production. Then we define a marginal income (immediate income for a given value of Q) by $MR = \frac{\mathrm{d}TR(Q)}{\mathrm{d}Q}$ In the majority of cases a discrete quantity can be replaced by a continuous one so the principle can be used for it too.

Similarly, marginal cost can be defined as a derivative of total cost with respect to production or flow of capital assets as a derivative of capital with respect to time.

6. Extremes

Extremes are probably the most important application of math in economics. It is very natural for economists to search for minimal costs or maximal profits. Generally we look for a global extreme because the independent variable is usually bounded either from above or from below or even from both sides. Then problems are formulated similarly to the following example:

Example. Total cost is defined by $TC(Q) = Q^2 - 20Q + 500$. For what number Q of products do the minimal and maximal cost occur and how much is the cost? Take into consideration that production Q can take values between 5 and 20 units only.

Solution. $TC'(Q) = 2Q - 20 = 0$, so $Q_0 = 10$ is a *stationary point*. Because $TC''(Q) = 2$ and $2 > 0$, there is a minimum in the $Q_0 = 10$. Its value is $TC(10) = 10^2 - 20 \cdot 10 + 500 = = 400$. Values in the border points are $TC(5) = 5^2 - 20 \cdot 5 + 500 = 425$, $TC(20) = = 20^2 - 20 \cdot 20 + 500 = 500$. So we can get maximum cost 500 units at $Q_1 = 20$ and minimum cost 400 at $Q_0 = 10$.

7. Differential of a function

The use of differential of a function is similar to the one used in mathematics. Supposing we know the value of a given quantity in the known fixed point we seek the change of the economic quantity when the independent variable changes a little. Then problems are formulated similarly to the following example:

Example. Total production cost is $TC(Q) = Q^2 - 20Q + 480$. Find both average and marginal costs generally. What are both average and marginal costs for the given production when production changes from $Q_0 = 12$ to $Q = 14$?

Solution. Generally: the average cost is $AC(Q) = \frac{TC(Q)}{Q} = Q - 20 + \frac{480}{Q}$. The change of the marginal cost is $\mathrm{d}TC(Q) = TC'(Q) \cdot \mathrm{d}Q$. For given values we have $AC(12) = 12 - 20 + 40 = = 32$ units. The differential is $\mathrm{d}TC(120) = (2 \cdot 12 - 20) \cdot (14 - 12) = 8$ units. This represents the linearized change of cost when production increases from 12 to 14. Because the original cost was $TC(12) = 12^2 - 20 \cdot 12 + 480 = 384$ units, the new cost is higher by 8, i.e. 392 units. For comparison we can get the absolute difference from $TC(14)-TC(12) = 14^2-20\cdot 14+480-384 = 12$. The apparent disproportion is explained by replacement of the original function by a straight line.

We learn from the submitted example that the change $\mathrm{d}Q$ must be kept sufficiently small not to bring a large error to the change of $TC(Q)$.

$$TC(Q) \cong TC(Q_0) + \mathrm{d}TC(Q_0) \cdot \mathrm{d}Q$$

The principle of approximation by addition the function value at a fixed point and its differential at the same point can be improved by addition of more Taylor series elements. For three Taylor series elements it can be

$$TC(Q) \cong TC(Q_0) + \mathrm{d}TC(Q_0) \cdot \mathrm{d}Q + \mathrm{d}^2TC(Q_0) \cdot \mathrm{d}Q^2$$

8. Definite integral

Taking into account that integration is the process similar to derivation but in the opposite direction and comparing it with the applications in the 5^{th} and 7^{th} sections we can consider that the well known Newton–Leibnitz formula

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

is suitable for the following relations:

change rate of a quantity $\longrightarrow$ total change of a quantity.

It represents the following relations in economics:

production intensity $\longrightarrow$ volume of production
marginal income $\longrightarrow$ total income
marginal cost $\longrightarrow$ total cost
flow of investment $\longrightarrow$ total investment

Then problems can be formulated similarly to the following examples:

Example. What is the total investment $I(t)$ from the end of the first to the end of the ninth year if the flow of investment is controlled by $i(t) = 3000 \cdot \sqrt{t}$.

Solution.

$$I(t) = \int_{t_1}^{t_2} i(t)\,\mathrm{d}t = \int_1^9 3000 \cdot \sqrt{t}\,\mathrm{d}t = 3000 \cdot \left[\frac{2}{3} \cdot \sqrt{t^3}\right]_1^9 = 54\,000 - 2\,000 = 52\,000 \text{ units.}$$

Example. Marginal income from using the equipment t years old is described by $z(t) = 10\cdot e^{-t}$. Find the total income Z and decide at what price is worth to buy the equipment supposing the cost is back in two years.

9. Conclusion

Submitted applications offer a set of standard options which is not closed to additional changes. The collection can be reduced or enlarged according to both specific requirements existing at the moment and objectives a teacher intends to implement. The emphasis is placed on pure economic applications but they can come from managerial or other related fields too.

We consider it very important that students meet the practical face of mathematics, because they will find a more familiar way to the subject and they will be able to use it in the future successfully.

References

EICHLER, Bohuslav, 1993. *Úvod do finanční matematiky.* Praha: Septima, 1993. ISBN 80–901446–4–0.

KOVAŘÍK, Pavel, 2010. *Matematika I.* Brno. VŠKE, učební text, 2010. ISBN 978–80–86710–25–9.

MEZNÍK, Ivan, 2001. *Matematika I.* Brno: Nakladatelství vědecké a odborné literatury Dočkal, 2001.

MEZNÍK, Ivan, 2001. *Matematika II.* Brno: Nakladatelství vědecké a odborné literatury Dočkal, 2001.

PŘÍVRATSKÁ, Jana, 2011. *Matematika II.* Brno. VŠKE, učební text, 2011. ISBN 978–80–86710–31–0.

RADOVÁ, Jarmila a Petr DVOŘÁK, 2001. *Finanční matematika pro každého.* 3. vyd. Praha: GRADA Publishing, 2001. ISBN 80–247–9015–7.

doc. RNDr. Pavel Kovařík, CS.
Karel Englis College, Inc.
Šujanovo náměstí 356/1, 602 00 Brno
E-mail: Pavel.Kovarik@vske.cz
Telefon: +420 543 211 506

RNDr. Rudolf Schwarz, CSc.
Karel Englis College, Inc.
Šujanovo náměstí 356/1, 602 00 Brno
E-mail: Rudolf.Schwarz@vske.cz
Telefon: +420 543 211 506